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5–7 Factoring Pattern for $x^2 + bx + c$, c positive

Objective: To factor quadratic trinomials whose quadratic coefficient is 1 and whose constant term is positive.

Vocabulary/Patterns

Factoring patterns for $x^2 + bx + c$ when c is positive:
When b is positive: $(x + ?)(x + ?)$
When b is negative: $(x - ?)(x - ?)$

Prime polynomial A polynomial with integral coefficients whose greatest monomial factor is 1 and which can't be written as a product of polynomials of lower degree. For example, $a^2 - 10a - 14$ is prime.

Example 1 Factor $x^2 + 6x + 8$.

Solution

1. The coefficient of the linear term is positive. The pattern is $(x + ?)(x + ?)$. List the positive factors of 8.

Factors of 8		Sum of the factors
1	8	9
2	4	6 ←

2. Find the pair of factors whose sum is 6: 4 and 2.
3. Therefore $x^2 + 6x + 8 = (x + 4)(x + 2)$.

You can check the result by multiplying $(x + 4)$ and $(x + 2)$.
$(x + 4)(x + 2) = x^2 + 2x + 4x + 8 = x^2 + 6x + 8$ √

Example 2 Factor $x^2 - 8x + 15$.

Solution

1. The coefficient of the linear term is negative. The pattern is $(x - ?)(x - ?)$. List the pairs of negative factors of 15.

Factors of 15		Sum of the factors
−1	−15	−16
−3	−5	−8 ←

2. Find the pair of factors whose sum is −8: −3 and −5.
3. Therefore $x^2 - 8x + 15 = (x - 3)(x - 5)$.

Factor. Check by multiplying the factors. If the polynomial is not factorable, write *prime*.

1. $x^2 + 4x + 3$ **$(x + 1)(x + 3)$**
2. $x^2 + 8x + 7$ **$(x + 1)(x + 7)$**
3. $c^2 - 9c + 14$ **$(c - 2)(c - 7)$**
4. $y^2 - 8y + 12$ **$(y - 2)(y - 6)$**
5. $r^2 - 5r + 6$ **$(r - 2)(r - 3)$**
6. $p^2 - 13p + 12$ **$(p - 1)(p - 12)$**
7. $q^2 + 15q + 14$ **$(q + 14)(q + 1)$**
8. $n^2 + 9n + 14$ **$(n + 2)(n + 7)$**
9. $a^2 - 13a + 22$ **$(a - 2)(a - 11)$**
10. $s^2 - 12s + 30$ **prime**
11. $x^2 + 18x + 32$ **$(x + 2)(x + 16)$**
12. $x^2 - 15x + 26$ **$(x - 2)(x - 13)$**

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Example 3 Factor $y^2 - 10y + 16$.

Solution

1. Since −10 is negative, think of the negative factors of 16 in your head. (After a little practice you will not need to write all the factors down.)
2. Select the factors of 16 with sum −10: −2 and −8.
3. Therefore $y^2 - 10y + 16 = (y - 2)(y - 8)$.

Factor. Check by multiplying the factors. If the polynomial is not factorable, write *prime*.

13. $a^2 + 10a + 30$ **prime**
14. $x^2 - 19x + 60$ **$(x - 4)(x - 15)$**
15. $k^2 - 21k + 54$ **$(k - 3)(k - 18)$**
16. $n^2 + 23n + 90$ **$(n + 5)(n + 18)$**
17. $k^2 - 10k + 21$ **$(k - 3)(k - 7)$**
18. $x^2 - 14x + 45$ **$(x - 5)(x - 9)$**
19. $k^2 + 7k + 12$ **$(k + 3)(k + 4)$**
20. $x^2 - 16x + 48$ **$(x - 4)(x - 12)$**
21. $a^2 - 11a + 20$ **prime**
22. $x^2 + 22x + 72$ **$(x + 4)(x + 18)$**
23. $72 - 17z + z^2$ **$(9 - z)(8 - z)$**
24. $20 - 12c + c^2$ **$(2 - c)(10 - c)$**
25. $54 - 15a + a^2$ **$(9 - a)(6 - a)$**
26. $63 - 16c + c^2$ **$(9 - c)(7 - c)$**

Example 4 Factor $x^2 - 12xy + 32y^2$.

Solution

$x^2 - 12xy + 32y^2 = (x - ?)(x - ?)$ Write the factoring pattern.
$= (x - 4y)(x - 8y)$ Fill in the negative factors of $32y^2$.

Factor. Check by multiplying the factors. If the polynomial is not factorable, write *prime*.

27. $x^2 - 11xy + 28y^2$ **$(x - 7y)(x - 4y)$**
28. $a^2 - 9ab + 18b^2$ **$(a - 3b)(a - 6b)$**
29. $c^2 - 18cd + 45d^2$ **$(c - 3d)(c - 15d)$**
30. $x^2 - 10xy + 21y^2$ **$(x - 3y)(x - 7y)$**
31. $c^2 - 14cd + 24d^2$ **$(c - 12d)(c - 2d)$**
32. $x^2 + 11xy + 30y^2$ **$(x + 6y)(x + 5y)$**
33. $y^2 - 16yz + 48z^2$ **$(y - 4z)(y - 12z)$**
34. $a^2 - 18ab + 45b^2$ **$(a - 3b)(a - 15b)$**
35. $d^2 + 10de + 24e^2$ **$(d + 4e)(d + 6e)$**
36. $y^2 - 27yz + 72z^2$ **$(y - 3z)(y - 24z)$**

Mixed Review Exercises

Solve.

1. $-12 + x = -7$ **{5}**
2. $d + (-4) = -9$ **{−5}**
3. $-12 + b = 13$ **{25}**
4. $a + 3 = |2 - 9|$ **{4}**
5. $17m = 68$ **{4}**
6. $3p + 15 = -60$ **{−25}**
7. $-\frac{1}{3}x = 9$ **{−27}**
8. $\frac{r}{2} - 3 = 6$ **{18}**
9. $-18x = 162$ **{−9}**